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BULK ARRIVAL RETRIAL QUEUE WITH FLUCTUATING MODES OF SERVICE, IMMEDIATE FEEDBACK, SERVER VACATION AND ORBITAL SEARCH

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ABSTRACT
This paper deals with single server retrial queue with fluctuating modes of service. Server provides M modes of service with different service rates. At the arrival epoch of a batch, if the server is idle, then one customer enters for service and others join the orbit. Otherwise all the customers join the orbit. After completion of service, the unsatisfied customers opt for re-service. At the completion epoch of each service, the server takes a single Bernoulli vacation. After vacation completion the server may search for customer in the orbit. The supplementary variables corresponding to retrial time, service time and vacation time are incorporated to determine the queue size distribution. The mean number of customers in the orbit, the mean number of customers in the system and system probabilities are obtained. Finally the stochastic decomposition law is verified. The analytical results are validated with the help of numerical illustrations.

KEYWORDS: Retrial Queue, Fluctuating Modes, Immediate Feedback, Vacation and Orbital Search.

INTRODUCTION
Queueing systems with repeated attempts are characterized by the phenomenon that a customer finding all the servers busy upon arrival is obliged to leave the service area and repeat the request for service after some random time. Retrial queues have been widely used to model many problems in telephone switching systems, telecommunication networks and computers competing to gain service from a central processor. The recent bibliographies on retrial queues can be found in Falin (1990), Falin and Templeton (1997) and Artalejo (1999, 2010).

Re-service is a real life phenomenon where customers receiving some kind of service may need to repeat or demand re-service for the service taken. Re-service was initially studied by Madan et al. (2004). Recently the concept of re-service has been studied by Tadj and Ke (2008), Kumar and Arumuganathan (2011) and Baruah et al. (2013).

In the retrial setup, each service is preceded and followed by the server's idle time because of the ignorance of the status of the server and orbital customers by each other. Server's idle time is reduced by the introduction of search of orbital customers immediately after a service completion. The search for customers immediately on termination of a service was first introduced in the classical queue by Neuts and Ramalhoto (1984). Orbital search after service completion have been investigated by Artalejo et al. (2002), Krishnamoorthy et al. (2005), Sumitha and Udaya Chandrika (2012) and Rajadurai et al. (2015).

In this paper we analyse Bulk arrival retrial queue with fluctuating modes of service, immediate feedback (re-service), server vacation and orbital search.

MODEL DESCRIPTION
Consider a single server retrial queueing system in which customers arrive in batches according to a compound Poisson process with rate λ. The batch size Y is a random variable with distribution function P(Y=k) $= C_k$, k=1,2,..., and probability generating function C(z) having first two moments m_1 and m_2. The server provides M heterogeneous modes of service and the probability of providing mode i service is $p_i (1 \le i \le M)$. If an arriving batch finds the server free, one of the customers in the batch begins any one of the M modes and the

rest join the orbit. Inter-retrial times have an arbitrary distribution function A(x), density function a(x), Laplace –Stieltje's transform $A^*(s)$ and conditional completion rate $\eta(x) = \frac{a(x)}{[1-A(x)]}$.

The service time of mode i (i= 1,2,…,M) follows a general distribution with distribution function $B_i(x)$, density function $b_i(x)$, Laplace–Stieltje's transform $B_i^*(s)$, n^{th} factorial moments $\mu_{i,n}$ and conditional completion rate $\mu_i(x) = \frac{b_i(x)}{[1-B_i(x)]}$.

At the completion of each service the server takes a single vacation with probability τ or waits for the next customer with complementary probability $1-\tau$. The vacation time is generally distributed with distribution function V(x), density function v(x), Laplace Stieltje's transform $V^*(s)$, n^{th} factorial moments v_n and conditional completion rate $\gamma(x) = \frac{v(x)}{[1-V(x)]}$.

After completion of mode I service, the customer may opt for the same service with probability r_i or leave the system with its complementary probability $(1- r_i)$. In this case it is assumed that the customers are allowed to repeat the service only once.

After the completion of vacation, if the orbit is non-empty the server searches for the customers in the orbit with probability θ or remains idle with probability $\bar{\theta}$.

STEADY STATE DISTRIBUTION

Let N (t) denote the number of customers in the orbit at time t and C(t) denote the state of the server defined as

$$C(t) = \begin{cases} 0, & \text{if the server is idle} \\ i, & \text{if the server is busy in mode i service} \\ M+i, & \text{if the server is busy in mode i re-service} \\ 2M+1, & \text{if the server is on vacation} \end{cases}$$

The state of the system at time t can be described by the Markov process $\{X(t); t \geq 0\} = \{C(t), N(t), \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t); t \geq 0\}$. If C(t)=0, then $\xi_0(t)$ represents the elapsed retrial time, if C(t)=i$(1 \leq i \leq M)$ $\xi_1(t)$ represents the elapsed service time, if C(t)=M+i$\xi_2(t)$ represents the elapsed re-service time and if C(t)=2M+1 $\xi_3(t)$ represents the elapsed vacation time.

Define the following probability densities

$I_0(t) = P\{C(t) = 0, N(t) = 0\}$

$I_n(x,t)dx = P\{C(t) = 0, N(t) = n, x \leq \xi(t) < x + dx\}$, $x \geq 0$, $n \geq 1$

$P_{i,n}(x,t)dx = P\{C(t) = i, N(t) = n, x \leq \xi(t) < x + dx\}$, $x \geq 0, n \geq 0$, i = 1,2,…,M.

$Q_{i,n}(x,t)dx = P\{C(t) = M+i, N(t) = n, x \leq \xi(t) < x + dx\}$, $x \geq 0, n \geq 0, i = 1,2, \ldots, M$.

$V_n(x,t)dx = P\{C(t) = 2M+1, N(t) = n, x \leq \xi(t) < x + dx\}$, $x \geq 0$, $n \geq 0$

Let I_0, $I_n(x)$, $P_{i,n}(x)$, $Q_{i,n}(x)$ and $V_n(x)$ be the steady state probabilities of $I_0(t)$, $I_n(x,t)$, $P_{i,n}(x,t)$, $Q_{i,n}(x,t)$ and $V_n(x,t)$, where $n \geq 0$, $x \geq 0$, i = 1,2,…,M.

The system of equilibrium equations governing the model is given below

$$\lambda I_0 = (1-\tau)\left[\sum_{i=1}^{M}\left((1-r_i)\int_0^\infty P_{i,0}(x)\mu_i(x)dx\right) + \int_0^\infty Q_{i,0}(x)\mu_i(x)dx\right] + \int_0^\infty V_0(x)\gamma(x)dx \quad (1)$$

$$\frac{d}{dx}I_n(x) = -(\lambda + \eta(x))I_n(x), \quad n \geq 1 \quad (2)$$

$$\frac{d}{dx}P_{i,n}(x) = -(\lambda+\mu_i(x))P_{i,n}(x)+\lambda\sum_{k=1}^{n}c_k P_{i,n-k}(x),\ n\geq 0,\ i=1,2,\ldots,M \qquad (3)$$

$$\frac{d}{dx}Q_{i,n}(x) = -(\lambda+\mu_i(x))Q_{i,n}(x)+\lambda\sum_{k=1}^{n}c_k Q_{i,n-k}(x),\ n\geq 0,\ i=1,2,\ldots,M \qquad (4)$$

$$\frac{d}{dx}V_n(x) = -(\lambda+\gamma(x))V_n(x)+\lambda\sum_{k=1}^{n}C_k V_{n-k}(x),\ n\geq 0 \qquad (5)$$

with boundary conditions

$$I_n(0) = (1-\tau)\left[\sum_{i=1}^{M}\left((1-r_i)\int_0^\infty P_{i,n}(x)\mu_i(x)dx\right)+\int_0^\infty Q_{i,n}(x)\mu_i(x)dx\right] + \bar{\theta}\int_0^\infty V_n(x)\gamma(x)dx,\ n\geq 1 \qquad (6)$$

$$P_{i,0}(0) = p_i\left[\lambda C_1 I_0+\int_0^\infty I_1(x)\eta(x)dx+\theta\int_0^\infty V_1(x)\gamma(x)dx\right],\ i=1,2,\ldots,M. \qquad (7)$$

$$P_{i,n}(0) = p_i\left[\begin{array}{l}\lambda C_{n+1}I_0+\int_0^\infty I_{n+1}(x)\eta(x)dx+\lambda\sum_{k=1}^{n}C_k\int_0^\infty I_{n-k+1}(x)dx\\ +\theta\int_0^\infty V_{n+1}(x)\gamma(x)dx\end{array}\right],\quad n\geq 1;\ i=1,2,\ldots,M. \qquad (8)$$

$$Q_{i,n}(0) = r_i\int_0^\infty P_{i,n}(x)\mu_i(x)dx,\ n\geq 1,\ i=1,2,\ldots,M \qquad (9)$$

$$V_n(0) = \tau\sum_{i=1}^{M}\left\{\left((1-r_i)\int_0^\infty P_{i,n}(x)\mu_i(x)dx\right)+\int_0^\infty Q_{i,n}(x)\mu_i(x)dx\right\},\ n\geq 1 \qquad (10)$$

The normalizing condition is

$$I_0+\sum_{n=1}^{\infty}\int_0^\infty I_n(x)dx+\sum_{n=0}^{\infty}\left[\sum_{i=1}^{M}\int_0^\infty P_{i,n}(x)dx+\int_0^\infty Q_{i,n}(x)dx\right]+\sum_{n=1}^{\infty}\int_0^\infty V_n(x)dx=1 \qquad (11)$$

Define the probability generating functions

$$I(x,z)=\sum_{n=1}^{\infty} I_n(x)\,z^n;\ \ P_i(x,z)=\sum_{n=0}^{\infty} P_{i,n}(x)\,z^n;\ Q_i(x,z)=\sum_{n=0}^{\infty} Q_{i,n}(x)\,z^n;\ \ i=1,2,...,M$$

$$\text{and } V(x,z)=\sum_{n=0}^{\infty} V_n(x)\,z^n$$

Multiplying equations (2) to (10) by z^n and summing over for all possible values of n, we obtain the following results

$$\left[\frac{d}{dx}+(\lambda+\eta(x))\right] I(x,z)=0 \tag{12}$$

$$\left[\frac{d}{dx}+\lambda(1-c(z))+\mu_i(x)\right] P_i(x,z)=0,\ \ i=1,2,...,M \tag{13}$$

$$\left[\frac{d}{dx}+\lambda(1-c(z))+\mu_i(x)\right] Q_i(x,z)=0\,,\ i=1,2,...,M \tag{14}$$

$$\left[\frac{d}{dx}+\lambda(1-c(z))+\gamma(x)\right] V(x,z)=0 \tag{15}$$

$$I(0,z) = (1-\tau)\left[\sum_{i=1}^{M}\left(\left(1-r_i\right)\int_0^{\infty} P_i(x,z)\mu_i(x)\,dx\right)+\int_0^{\infty} Q_i(x,z)\mu_i(x)\,dx\right]$$
$$+\bar{\theta}\int_0^{\infty} V(x,z)\,\gamma(x)dx-\lambda I_0 \tag{16}$$

$$P_i(0,z) = \frac{p_i}{z}\left[\begin{matrix}\int_0^{\infty} I(x,z)\eta(x)\,dx+\lambda C(z)\left[\int_0^{\infty} I(x,z)dx+I_0\right]+\\ \theta\int_0^{\infty} V(x,z)\gamma(x)dx\end{matrix}\right], i=1,2,...,M. \tag{17}$$

$$Q_i(0,z)= r_i\,P_i(0,z)B_i^*(h(z)) \tag{18}$$

$$V(0,z) = \tau\left\{\sum_{i=1}^{M} P_i(0,z)\,B_i^*(h(z))\left[1-r_i+r_i\,B_i^*(h(z))\right]\right\} \tag{19}$$

where

$$h(z) = \lambda-\lambda c(z)$$

Solving the partial differential equations (12) to (15), we get

$$I(x,z) = I(0,z)\,e^{-\lambda x}(1-A(x)) \tag{20}$$

$$P_i(x,z) = P_i(0,z)\,e^{-\left[\lambda(1-c(z))\right]x}\left(1-B_i(x)\right) \tag{21}$$

$$Q_i(x,z) = Q_i(0,z)\,e^{-\left[\lambda(1-c(z))\right]x}\left(1-B_i(x)\right) \tag{22}$$

$$V(x,z) = V(0,z)\,e^{-\left[\lambda(1-c(z))\right]x}(1-V(x)) \tag{23}$$

Using equations (17), (18), (19), (21), (22) and (23) in equation (16) and simplifying, we get

$$I(0,z)=\lambda I_0\left[C(z)T_1(z)\left(1-\tau+\bar{\theta}\,\tau V^*(h(z))\right)-\left(z-\theta\,\tau V^*(h(z))T_1(z)\right)\right]\Big/D(z) \tag{24}$$

where

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$$T_1(z) = \sum_{i=1}^{M} p_i B_i^*(h(z))\left(1-r_i+r_i B_i^*(h(z))\right)$$

$$D(z)=z-\theta\,\tau V^*(h(z))T_1(z)-\left[A^*(\lambda)+C(z)\left(1-A^*(\lambda)\right)\right] T_1(z)\left(1-\tau+\bar{\theta}\tau V^*(h(z))\right)$$

Using equation (24), the equation (17) becomes

$$P_i(0,z) = \lambda I_0 A^*(\lambda) p_i [c(z)-1]\Big/ D(z),\ i=1,2,\ldots,M \tag{25}$$

Inserting equation (25) in equation (18) and (19), we get

$$Q_i(0,z) = \lambda I_0 A^*(\lambda) p_i r_i B_i^*(h(z)) [c(z)-1]\Big/ D(z),\ i=1,2,\ldots,M \tag{26}$$

$$V(0,z)=\lambda I_0 A^*(\lambda) (C(z)-1)\,\tau\, T_1(z)\Big/D(z) \tag{27}$$

Substituting the expressions of $I(0,z), P_i(0,z), Q_i(0,z)$ and $V(0,z)$ in equations (20), (21), (22) and (23), we get the following results

$$I(x,z)=\lambda I_0\left[C(z)T_1(z)\left(1-\tau+\bar{\theta}\tau V^*(h(z))\right)-\left(z-\theta\,\tau V^*(h(z))\,T_1(z)\right)\right]e^{-\lambda x[1-A(x)]}\Big/D(z) \tag{28}$$

$$P_i(x,z) = \lambda I_0 A^*(\lambda) p_i [C(z)-1]e^{-(h(z))x}\left[1-B_i(x)\right]\Big/D(z),\ i=1,2,\ldots,M \tag{29}$$

$$Q_i(x,z)=\lambda I_0 A^*(\lambda) r_i p_i B_i^*(h(z))[C(z)-1]e^{-(h(z))x}\left[1-B_i(x)\right]\Big/D(z),\ i=1,2,\ldots,M \tag{30}$$

$$V(x,z)=\lambda I_0 A^*(\lambda) (C(z)-1)\,\tau\, T_1(z)\,e^{-(h(z))x}\left[1-V(x)\right]\Big/D(z) \tag{31}$$

The partial probability generating function of the orbit size when the server is idle is given by

$$I(z) = \int_0^\infty I(x,z)dx$$

$$= I_0\left(1-A^*(\lambda)\right)\left[C(z)T_1(z)\left(1-\tau+\bar{\theta}\tau V^*(h(z))\right)-\left(z-\theta\,\tau V^*(h(z))\,T_1(z)\right)\right]\Big/D(z) \tag{32}$$

The partial probability generating function of the orbit size when the server is busy is given by

$$B(z) = \sum_{i=1}^{M}[P_i(z) + Q_i(z)]$$

$$= I_0 A^*(\lambda) \sum_{i=1}^{M} p_i\left(B_i^*(h(z))-1\right)\left[1+r_i B_i^*(h(z))\right]\Big/D(z),\ i=1,2,\ldots,M \tag{33}$$

The partial probability generating function of the orbit size when the server is on vacation is given by

$$V(z)=\int_0^\infty V(x,z)\,dx$$

$$=I_0 A^*(\lambda)\,\tau\, T_1(z)\left[V^*(h(z))-1\right]\Big/D(z) \tag{34}$$

Probability generating function of the number of customers in the orbit is given by

$$P_q(z)=I_0+I(z)+\sum_{i=1}^{M} P_i(z)+\sum_{i=1}^{M} Q_i(z)$$

$$= I_0 A^*(\lambda)[z-1]\Big/D(z) \tag{35}$$

Probability generating function of the number of customers in the system is given by

$$P_s(z) = I_0 + I(z) + z\sum_{i=1}^{M} P_i(z) + z\sum_{i=1}^{M} Q_i(z)$$
$$= I_0 A^*(\lambda)[z-1]\, T_1(z) \big/ D(z) \qquad (36)$$

PERFORMANCEV MEASURES

- Probability that the server is idle is given by
  $I = \lim_{z\to 1} I(z)$

$$= I_0\left(1 - A^*(\lambda)\right)\left[m_1 - m_1\theta\tau - 1 + \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i + 1\right) + \tau\lambda m_1 v_1 \sum_{i=1}^{M} p_i\right] \Big/ D'(1) \qquad (37)$$

where

$$D'(1) = 1 - m_1\left(1 - A^*(\lambda)\right)(1-\theta\tau) - \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i + 1\right) - \tau\lambda m_1 v_1 \sum_{i=1}^{M} p_i$$

- Probability that the server is busy is given by
  $B = \lim_{Z\to 1} B(z)$

$$= I_0 A^*(\lambda)\,\lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(1 + r_i\right) \Big/ D'(1) \qquad (38)$$

- Probability that the server is on vacation is given by
  $V = \lim_{z\to 1} V(z)$

$$= I_0 A^*(\lambda)\,\tau\,\lambda m_1 v_1 \sum_{i=1}^{M} p_i \Big/ D'(1) \qquad (39)$$

Using normalizing condition I_0 is obtained as

$$I_0 = \frac{1 - m_1\left(1 - A^*(\lambda)\right)(1-\theta\tau) - \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i + 1\right) - \tau\lambda m_1 v_1 \sum_{i=1}^{M} p_i}{A^*(\lambda)} \qquad (40)$$

Mean number of customers in the orbit L_q under steady state condition is given by

$$L_q = \lim_{z\to 1} \frac{d}{dz} P_q(z)$$
$$= \frac{D'(1)N''(1) - N'(1)D''(1)}{2D'(1)^2} \qquad (41)$$

where N(z) and D(z) are the Numerator and Denominator of $P_q(z)$.

$$N'(1) = I_0 A^*(\lambda)$$

$$D''(1) = -m_2\left(1 - A^*(\lambda)\right)(1-\theta\tau) - 2\tau\lambda m_1^2 v_1\left(1 - A^*(\lambda)\right) - \bar{\theta}\sum_{i=1}^{M} p_i$$
$$-2\lambda^2 m_1^2 \sum_{i=1}^{M} p_i \mu_{i1}^2 r_i - \lambda^2 m_1^2 \sum_{i=1}^{M} p_i \mu_{i2}\left(r_i + 1\right) - \lambda m_2 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i + 1\right)$$
$$-\tau\lambda^2 m_1^2 v_2 \sum_{i=1}^{M} p_i - \tau\lambda m_2 v_1 \sum_{i=1}^{M} p_i - 2\lambda m_1^2\left(1 - A^*(\lambda)\right)(1-\theta\tau)\sum_{i=1}^{M} p_i$$
$$\mu_{i1}\left(r_i + 1\right) - 2\lambda^2 m_1^2 \tau v_1 \sum_{i=1}^{M} p_i \mu_{i1} - 2\lambda^2 m_1^2 v_1 \sum_{i=1}^{M} p_i \mu_{i1} r_i$$

The mean number of customers in the system is given by

$$L_s = \lim_{z \to 1} \frac{d}{dz} P_s(z)$$
$$= L_q + B \quad (42)$$

STOCHASTIC DECOMPOSITION

Theorem:

The number of customers in the system (L_s) under steady state can be expressed as the sum of two independent random variables one of which is the mean number of customers (L) in the classical batch arrival queueing system with fluctuating modes of service, immediate feedback and orbital search and other is the mean number of customers in the orbit (L_I) given that the server is idle or on vacation.

Proof:

The probability generating function, π (z) of the system size in the classical batch arrival queue with fluctuating modes of service, immediate feedback and orbital search is given by

$$\pi(z) = \frac{\left(1 - \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1} \left(r_i + 1\right)\right) [(z-1)\ T_1(z)]}{z - T_1(z)} \quad (43)$$

The probability generating function, $\psi(z)$ of the number of customers in the orbit when the system is idle or on vacation is given by

$$\psi(z) = \frac{I_0 + I(z) + V(z)}{I_0 + I + V}$$

$$= \frac{[z - T_1(z)]\, D'(1)}{\left[1 - \lambda m_1 \sum_{i=1}^{M} p_i \mu_{i1}\left(r_i + 1\right)\right] D(z)} \quad (44)$$

From equations (36),(43) and (44) ,we see that

$$P_s(z) = \pi(z).\ \psi(z) \quad (45)$$

Differentiating (45) with respect to z and taking limit as z→1, we get

$$L_s = L + L_I$$

NUMERICAL RESULTS

Assume that the retrial time, service time and vacation time follow exponential distribution with parameters η, μ_1, μ_2 and γ. respectively.

Numerical analysis are carried out with stability condition by setting λ = 0.3, η = 5, μ_1 = 12, μ_2 = 13, r_1 = 0.4, r_2 = 0.4, p_1 = 0.7, p_2 = 0.3, m_1 = 1.5, m_2 = 1, γ = 7,θ = 0.6.

The influence of parameters (λ, μ_1), (p_1,τ) and (η,θ) on the performance measures I_0+I and L_s are displayed in Fig.4.1 to 4.3.

From the figures it is observed that

- Probability I_0+I that the server is idle, decreases for increasing values of λ, p_1 and τ, increases for μ_1 and independent of η and θ
- Mean number of customers in the system L_s increases for increasing values of λ, p_1 and τ and decreases for μ_1 ,η and θ

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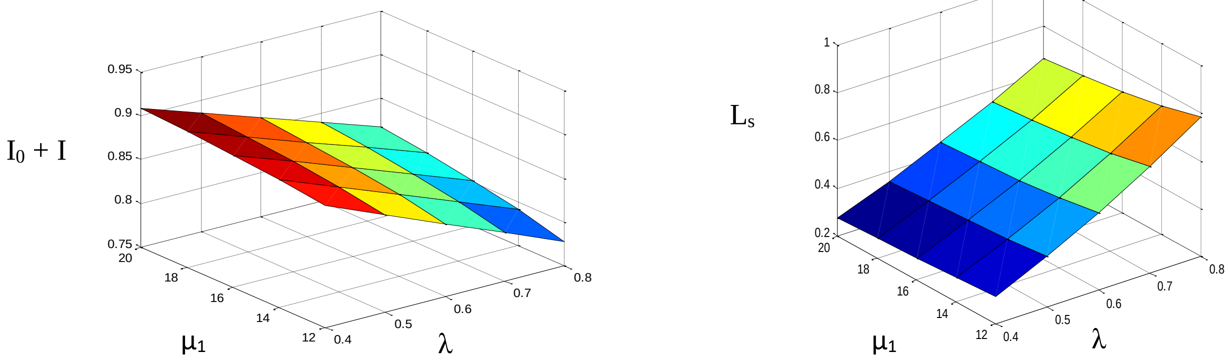

Fig.4.1 Effect of (λ, μ_1) on I_0+I and L_s

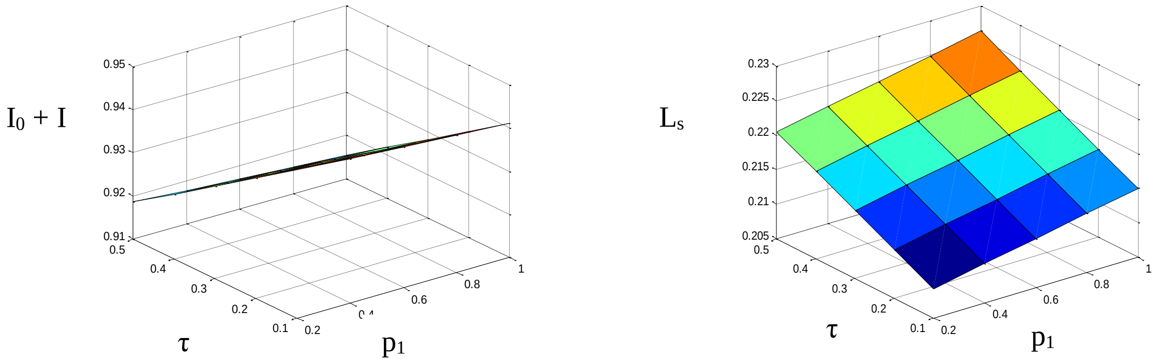

Fig.4.2 Effect of (p_1, τ) on I_0+I and L_s

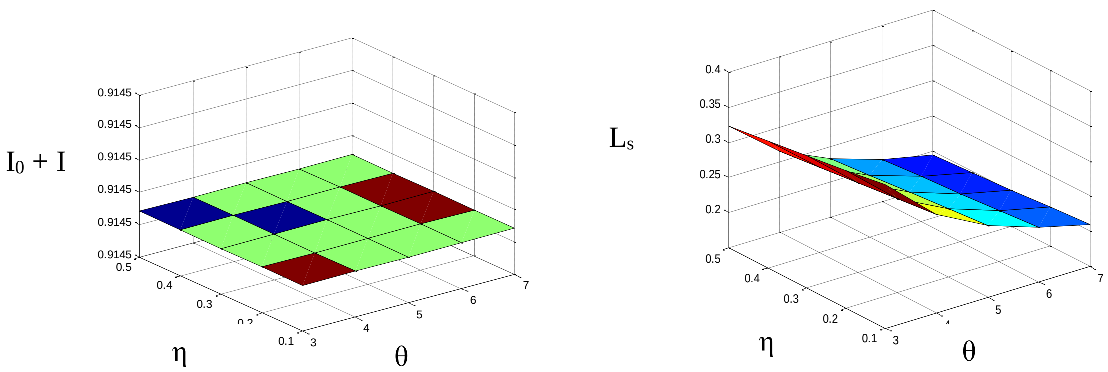

Fig.4.3 Effect of (η , θ) on I_0+I and L_s

CONCLUSION

A single server bulk arrival retrial queueing system with fluctuating modes of service, immediate feedback, server vacation and search of customers from the orbit is studied. Explicit results for the probability generating functions and other steady state system performance measures are derived. Numerical analysis is performed.

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